THE FUTURE DEVELOPMENT OF MATHEMATICAL EDUCATION[1]

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You have already heard this afternoon of the work of two organizations that have been and are actively interested in the improvement of mathematical education. You have also had presented to you programs for adapting mathematical instruction to the needs of two rather recent types of school organization. It is apparent from this afternoon's discussion alone that the teachers of mathematics here and throughout the country are alive to their opportunities and their responsibilities. They realize the great service to society which they can perform by selecting from the vast store of mathematical knowledge those elementary methods and processes that are of widest use in the modern world, organizing them into coherent courses, and presenting them effectively to their classes. It is apparent to the careful observer that existing mathematical courses have not been constructed with due regard to the relative importance for the general student of the different mathematical methods and principles that are available for instruction in school and college. Our courses have been arranged primarily for the benefit of those who will continue their mathematical education. That they do contain much material of great use to the general student arises from the fortunate circumstances that most of the processes in elementary mathematics have some important applications in the world of to-day. But the conscientious teacher of mathematics will not allow his good fortune in this respect to paralyze his initiative. If we can add considerably to the usefulness of our courses by reorganizing them, by all means let us do it. It is certainly our duty and it should be our pleasure.

If we consider the various individuals who in the past have contributed to the world's progress, we find that we can group them into three principal classes. We have first the faithful plodder who sees small improvements that can be made here and

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there and contributes his best energies to making them. We owe much to the combined efforts of the plodders and we should have proper appreciation for them. Their chief fault is that at times through their lack of vision they sacrifice the greater good to the lesser. The next group is composed of the dreamers or idealists. They are animated by a great vision of things as they ought to be, but they are blind to practical difficulties, and for this reason they rarely succeed in bringing about by their own efforts the reforms they advocate. But they do serve to galvanize the indifferent by their personal enthusiasm, and in this way they frequently make it possible for other men to carry through to success the tasks in which they have failed. In spite of their limitations, let us render due honor to the idealists. The third group comprises those persons who combine the qualities of the idealists and the plodders. They have the fervent enthusiasm which comes from the vision of a great ideal, but they have also the practical common sense to realize that we may never reach that ideal and that at best we must proceed toward it by steps that are slow but sure. This is the most useful group of the three, and we should all strive to belong to it. We should have high ideals, because they furnish enthusiasm in our work and guidance of our efforts, but we should also have the patience and courage to work our way toward them through the maze of practical difficulties in which we inevitably find ourselves.

Since teachers of mathematics are at present so generally occupied with the problem of improving mathematical education, and since furthermore teachers are more apt to be idealists than the generality of mankind, it seems fair to conclude that many teachers of mathematics have more or less definite ideals as to what mathematical education ought to be. Granting that this is the case, it would seem desirable that all of us who have such ideals should endeavor to formulate them so that by comparison and discussion we may come to approximate agreement as to our larger aims. With this idea in mind, I am going to present briefly my own views as to the ideals which should govern the future development of mathematical education.

We who have been long engaged in the study and teaching of mathematics realize full well that it is one of the most essential elements in the warp and woof of our modern civilization. We know that most of our commerce, industry, and science depends to a greater or less extent on mathematical knowledge, and that many of the arts owe a considerable debt to this fundamental subject. We are quite aware that the complete wiping out of every vestige of mathematical knowledge would immediately paralyze the activities of the civilized world. But to how many of our students, even the more gifted ones, do we bring home this realization? To how many do we give just a glimpse of the amazing vitality and power of a living and growing mathematical science?

In every great commercial activity of the modern world much attention is paid to the sales force. It is not enough to produce a worthy article; you must persuade the buying public that it is a worthy article. I believe that we mathematicians and teachers of mathematics have in the past been woefully negligent of the selling end of our activities. We have produced mathematical knowledge in enormous quantities; such of it as the rest of the world has been wise enough to utilize has been exceedingly beneficial. We have handed this knowledge over the counter very cheerfully to such students and inquirers as came to seek for it. But in general we have made no systematic and intensive effort to persuade our students and fellow citizens of the value of mathematical knowledge. By selling activities in connection with mathematics and the teaching of mathematics I of course do not mean selling in the ordinary commercial sense. What we must aim to secure for mathematics is not mere dollars and cents, but an appreciation of its value and an enthusiasm in its pursuit. Adequate material support will follow as a natural consequence.

The important question then is what shall we do to make people in general and particularly our students realize the value of mathematical education? We must of course organize our courses so as to include as many as possible of those elements of mathematical knowledge that are of real importance in the world of today. We must eliminate from elementary courses

difficulties that are purely artificial and which do not arise in the ordinary applications of mathematics. There are plenty of real difficulties to include in our courses; we can well dispense with the artificial ones.

Such topics as we do decide to teach on the basis of the above principles should be presented in close connection with some of their important applications. The student who is learning mathematics should be made to realize that he is acquiring knowledge that real live people find it highly necessary to use in human activities of fundamental importance.

All this is easy to say. It is by no means easy to do. As I told you at the beginning, I am describing an ideal, a vision. In choosing applications to reinforce the teaching of mathematics, one must proceed with great care. The application may be so technical as to be more difficult to understand than the mathematical principle on which it is based. We cannot at the same time study mathematics and a multitude of other difficult subjects. It should be the business of all earnest teachers of mathematics to keep a constant lookout for simple applications of elementary mathematics that are of importance in the modern world and which at the same time can be readily comprehended by the average student. Some of the more technical applications may be referred to and described in a general way, but the applications that drive home the principle should be as simple as possible.

In addition to bringing home to the student the wide use of mathematical knowledge in the activities of the modern world, we must also give him some notion of its origin and growth and its important role in the development of our civilization. We must not let him rest under the impression that mathematics was invented in order to provide intricate and vexatious puzzles for the adolescent mind. We must demonstrate to him that man was led to the pursuit of mathematical knowledge by his eager desire to understand the universe and to control the forces of nature, that he found this knowledge essential for the higher developments of trade and commerce and all of the other varied developments that have had a place in the creation of

our present day civilization, in short that the progress of the world is now and always has been bound up with the development of our knowledge of mathematics.

We must draw from biography as well as history in our endeavors to create in our students a real enthusiasm for mathematics. I believe the average young American is almost if not quite as ready to applaud mental achievement of a high order as he is to applaud prowess in the world of sport, provided only that the former is properly presented to him. I do not see why the tenacity of purpose and the prodigious mental exertion by means of which the great heroes of our science scaled hitherto impregnable heights of mathematical theory should not arouse something of the same thrill as the description of the physical endurance and courage displayed in the ascent of the Matterhorn or the dash to the North Pole. Let us make plain to our disciples that the quest for mathematical knowledge is one of the most important and most fascinating portions of the great adventure, of man's eternal effort to penetrate further into unknown regions and master them for his possession and use, and they will be convinced that while mathematics may be a difficult subject it never can be a dry subject.

I know this is a large program that I have here mapped out, but I think it is an entirely possible program if we adopt it for our own and put forth our best efforts to realize it. These efforts should of course include much self-development. He who would create enthusiasm in others must first have enthusiasm of his own, and it is difficult to remain enthusiastic about a subject in which we are not steadily growing. A complete knowledge of all the mathematics thus far discovered is not possible, even for the most gifted mathematician of the age, so there is nothing to limit our advance in the science, even though we are not ourselves engaged in investigation. I consider it one of the most important duties of every teacher of mathematics to be constantly increasing his stock of mathematical knowledge. He should in particular study the history of his subject and its intimate relationship with the history of other sciences, and the progress of civilization. He should be familiar with the names and the principal achievements of the great leaders of mathematical thought, past and present.

Finally the teacher of mathematics should never allow himself to become static in regard to his work. No matter how much we may improve the teaching of mathematics, something will always remain to be done. We who deal so constantly with variable quantities have no license to forget that the world about us is in an eternal state of flux and that we must ever adjust ourselves to changing conditions. In our work just as in all other of the world's activities, it is essential to remember that

"New occasions teach new duties; time makes ancient
good uncouth;
He must upward still and onward who would keep
abreast with truth."